Physics 303K Handout 7

1. **Second Midterm**

   The second midterm will be held on Wednesday, March 10 at 7-9pm in WEL 2.224. The test will cover Chapters 6-9 in Resnick, Halliday, and Krane. Exactly how much of Chapter 9 will be on the test depends on how far Dr. Turner gets in lecture.

2. **Chapter 7: Systems of Particles**

   Before now, we have treated objects as points in space. This is fine if the particle just undergoes translation, because every part of the object moves in the same way. However, this simplification breaks down when the body starts to rotate. When a body rotates, different parts of the object move in different ways. Our job is now to make this more exact.

   (a) *Definition of System* A system is a collection of objects that is distinguished from its surrounding environment. Forces between objects within the system are *internal forces.* Forces between an object inside the system and an object outside the system are *external forces.*

   (b) *Center of Mass* The center of mass is a unique point of a system that undergoes motion like a point particle regardless of the system's rotation. (This is like the baton in Resnick pg 140.) You can think of the center of mass as the 'balanced point'. In a reference frame following the center of mass, each point *only rotates* around the center of mass (i.e. There is no longer a translational component along with the rotation.) We can calculate the center of mass $\vec{r}_{cm}$ via:

   $$\vec{r}_{cm} = \frac{\sum_n m_n \vec{r}_n}{M}, \tag{1}$$

   where $M = \sum_n m_n$. Remember that the vector equation above is really three equations, one for each spatial dimension:

   $$x_{cm} = \frac{\sum_n m_n x_n}{M}, \quad y_{cm} = \frac{\sum_n m_n y_n}{M}, \quad z_{cm} = \frac{\sum_n m_n z_n}{M}. \tag{2}$$

   For a continuous system of mass, the above summation needs to become an integral:

   $$\vec{r}_{cm} = \frac{1}{M} \int \vec{r} dm. \tag{3}$$

   The velocity and acceleration of the center of mass is simply

   $$\vec{v}_{cm} = \frac{1}{M} \int \vec{v}\, dm, \quad \vec{a}_{cm} = \frac{1}{M} \int \vec{a}\, dm. \tag{4}$$

   The center of mass obeys all the usual kinematic equations.

   (c) *Linear momentum is still conserved.* The linear momentum of a system S is

   $$\vec{P} = M \vec{v}_{cm}. \tag{5}$$

   Newton's second law for the system is

   $$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}. \tag{6}$$

   (d) *Variable Mass* The last equation can be rewritten as

   $$\sum \vec{F}_{ext} = M \vec{a}_{cm} + \vec{v}_{cm} \frac{dM}{dt}. \tag{7}$$

3. **Beginning of Chapter 8**

   (a) *Polar Coordinates* Instead of Cartesian coordinates (x, y, z), it is convenient to use polar coordinates (r, θ, z). We usually orient the z-direction along the axis of rotation.

   (b) *Right Hand Rule* We use the convention that counterclockwise motion is the positive sense of rotation. The Right Hand Rule (RHR) lets us remember which way is positive.

   (c) *Rotational Kinematics* The equations of motion for rotation are the same as for translation with the transform $(x, v, a) \rightarrow (\theta, \omega, \alpha)$. The relationship between angular position θ, angular velocity ω, and angular acceleration α are

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}. \tag{8}$$

   Assuming constant angular acceleration α, The kinematic equations are found by integration

$$\omega = \alpha t + \omega_0 \tag{9}$$

$$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \tag{10}$$

   and

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0). \tag{11}$$

   With the equations so similar what is fundamentally different about translation and rotation? In short, $\theta + 2n\pi = \theta$ for any integer n, while $x + L = x$ only for $L = 0$. You can relate linear kinematic variables to the corresponding angular variable:

$$\theta r = s, \ \omega r = v_t, \ \alpha r = a_t. \tag{12}$$

4. **Little Bit of Chapter 9** Torque is $\vec{\tau} = \vec{r} \times \vec{F} = rFsin(\theta)$. Use the RHR for the cross product. Newton's second law for torque is $\sum \vec{\tau}_{ext} = I\alpha$. The moment of inertia $I = \int r^2 dm$.

5. **Keeping Things Whole**
   Mark Strand

   In a field
   I am the absence
   of field.

   This is
   always the case.
   Wherever I am
   I am what is missing.

   When I walk
   I part the air
   and always
   the air moves in
   to fill the spaces
   where my body's been.

   We all have reasons
   for moving.
   I move
   to keep things whole.

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